The Chains of Life[1]
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Praeludium

We are here to discuss the notion of *Lebensform*---"life form", "life-form" or "form of life"---in Wittgenstein. I shall stress *form* in a logical sense, as well as *life*. The idea is to inflect *Lebensformen* toward *logic*: possibilities (and necessities) of life structuring, possibilities and necessities (forms) of action, possibilities (and necessities) of shapings (in that sense of "form") of life. My metaphorical quarry, the thing I shall try to unpack and unfold, is Wittgenstein's treatment of the idea of *chains* in logic, which are chains of— and in—life: living chains. Hence my title.

There is no need for me to discuss *of*. First, it's not in the German *Lebensform*. Second, intentionality and/or the idea of life *as* formed are large topics, and others will do better with these than I can. Third, I don't really know how to translate *Lebensform* well into English.[2] It's a complicated term with a complicated history in German that others have discussed. I will focus on what *Wittgenstein* was doing with the fusion of the terms that is new and idiosyncratic.

It is a question how unified Wittgenstein's uses of the term "*Lebensform*" are, and how important the notion is for his work. It is not a "pivotal" concept for him, according to Hacker,

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[1] For work on sources and ideas I have relied, as so many of us do, on many other people. First and foremost, there is Wittgenstein (1999), coupled with Biggs and Pichler (1993). For analysis of the evolution of usages, I am also indebted to von Wedelstaedt (2007), given to me by Felix Mühlhölzer, to whom I am thereby indebted---also for our many discussions. von Wedelstaedt persuasively groups the uses of *Lebensform* into 12 distinct ones, with characterization of the genealogies in Wittgenstein's writings. Inspiration for my historical remarks has derived from Kanamori (manuscript) and discussions with Mauro Engelmann, Sanford Shieh, Jonathan Smith, Akihiro Kanamori, Max Weiss, and the students in my spring 2016 Seminar on Wittgenstein at Boston University. Every error is mine.

[2] In the Wittgenstein (2009) updated translation of PI, Hacker and Schulte opt for "life-form" sometimes, and "form of life" other times. The hyphen is ugly, though perhaps necessary, and maybe preferable to "lifeform", though I'm not sure. The OED tells us (3rd ed. 2009) that the first usage of "life-form" was 1850, in the *Quarterly Homoeopathic Journal* ("The contagion must have had an origin similar to that of the inferior life-forms"; after that the influence of Darwin is prevalent, both involve ranking of life forms. The two root meanings are, in the OED: 1. A living organism and 2. (In Botany or Ecology) "a habit of growth or vegetative form characteristic of a plant or group of plants, esp. as representing adaptation to the environment". Habits, characteristics (*Merkmale*) and adaptations need not be ranked, I suppose, even if they may be linked.

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but a "minor signpost" of a major shift in "philosophy, philosophy of language and logic, and philosophy of mathematics that Wittgenstein instigated".[3] In some sense I agree. Yet I think *Lebensform* is doing different, distinctive, fundamental work of its own in Wittgenstein, and that the ordering of his argumentation has been lost in the grand characterization Hacker gives us. Certainly we cannot measure the importance of this concept by counting or locating its occurrences and history, much less by reaching toward Wittgenstein as lone instigator.

Lebensform is an important index of the development of Wittgenstein's thought, representing, from a regressive point of view, the place he finally chased down *to*. In foundational matters such as these, it is often the simplest, naïve perspective, the working through of how to get back home to the primordial basis of *simplicity*, the extraction of fundamentals from a jungle of technical accretions, that comes *last* in a thinker's evolution. This chronological evolution tends to mirror certain logical necessities of the regressive method. It certainly characterizes repeatedly the history of the mathematicization of logic between Dedekind's *Habilitationsschrift* of 1854, and Turing's final paper "Solvable and Unsolvable Problems" of 1954, what Sieg has—rightly I think—called a *centuria mirabilis* in logic and the foundations of mathematics.[4] Here I will be chaining Wittgenstein to certain elements of that tradition, as I think he himself did.

It took Wittgenstein a long time to get to *Lebensform*. When he did, it marked a crucial juncture in his thought, the period 1936-7. This was the period when he generated the first two drafts of the *Investigations* and broke through to his later, mature style of writing. *Lebensform* enters his writing then.

The first occurrence of *Lebensform* is in MS 115, p. 239, from 1936, his attempted translation and revision of *The Brown Book*, undertaken in earnest in the fall of 1936, in Norway, and abandoned by November. It occurs in the midst of a struggle over imagining a use of "language/*eine Sprache*/*Lebensform*/*Form des Lebens*" (there are several variants here) in which one could think securing of a "*gap*" [*Kluft*] between dark red and light red.[5] He soon wrote on p.

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[3] Hacker (2015), p. 1.
[4] Cf. Sieg (2013), p. 8, philosophically contextualized in Floyd (2013), p. 1004.
[5] MS 115, page 239, EPB 108A, cf. WA V, 202:
10[7|8] Umgekehrt könnte ich mir auch einen Sprachgebrauch «eine Sprache» (& das heißt wieder eine Lebensform «Form des Lebens») denken, der «die» zwischen Dunkelblau«rot» & Hellblau«rot» eine Kluft befestigt. etc.

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292, however, that "This whole 'attempted revision' from page 118 to here is *worthless*". And thereafter he faced a tremendous intellectual and personal crisis.

However, that same dark month, alone in Norway, filled with confessions and self-crises, he began again anew. He drafted what would become, over the course of spring 1937, the beginning of the origin of the *Investigations*, the so-called *Urfassung*.[6] He commenced in November 1936 with MS 142. Its first 76 pages or so were composed before Christmas in 1936, the rest presumably written in the spring of 1937; the whole covers roughly the substrata of *Investigations* §§1-189(a). It ends with his calling the question from the interlocutor about the steps *not* being "determined" by the formula one that "contains a mistake".[7]

When he returned to Britain in summer 1937 he had a typescript made (TS 220) and showed it to G.E. Moore and some other colleagues, according to Rhees.[8] TS 220 contains the origin of PI §§18-23, with "*Lebensform*" occurring in the source of §19 (the imagined language of only orders in battle, and countless other things) and that of §23 ("'language-*game*' is meant to emphasize the fact that the *speaking* of language is part of an activity, or of a *Lebensform*").

Rhees reports Moore saying to him that Wittgenstein told him in the summer of 1937 that *The Brown Book* had used "the false method", but now he had gotten to the "correct method". Moore said he did not know what Wittgenstein meant.[9] But we can guess, with the power of hindsight.

A crucial earmark of MS 142's importance is that it is here, for the first time, that Wittgenstein thematizes *simplicity* in the manner of his mature work. He adds for the first time the swatch of remarks beginning with the quotation from Socrates's dream in the *Theaetetus,* remarks he models with a language-game with arrangements of colored squares (PI §§46-76, these precede the remark about "family resemblances"). This swatch ends with the question, "In what way is logic sublime?" (cf. PI §89). He was able to go on through the spring of 1937 implementing what Engelmann calls the "genetic" method: the dialectical juxtaposition of his mature thinking with his *Tractatus* thinking, to signal effect.[10]

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[6] For a critical edition of these versions see Wittgenstein (2001).

[7] In Floyd (1991) I analyze this context line by line, with attention to the manuscript composition.

[8] Rhees wrote (EPB, pp. 12-13) that the manuscript included TS 221 and that Moore gave this manuscript to him to read in the spring of 1938. But the editors of the Wittgenstein edition of PI disagree ((2009), xviii ff.), saying that Wittgenstein only had TS 220 in Cambridge with him in the summer of 1937.

[9] EPB 12-13.

[10] See Englemann (2013) for a discussion of this new method.

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In the *Tractatus*, the simples are *absolute* ideals, absolute "indefinables".[11] In the middle period, simplicity was conceived to be relative to a *Satzsystem* or, by the time of *The Blue Book*, to a language-game, and Wittgenstein wrote of "undefined" and "unanalyzed" notions. It is only by 1936-7 that Wittgenstein gets where he needs to go with simplicity, is able to embrace the most radical idea, the idea that simplicity is, not simply relative to a system, but fluid. Necessarily and absolutely fluid. Necessarily and absolutely a matter of ongoing discussion. For the very sake of exactness. We always have to start somewhere: there are always "simples", we always take a step from somewhere. But each step may be unwound as itself complex, moved, critically reflected upon, contested. And then that contestation itself, with its simples, may be scrutinized.

It was at precisely this point in time, in 1936-7, that Wittgenstein surrendered the mode of proceeding in his writing step-by-step, in a serial or linear manner beginning from a single starting point, as he had done in *The Blue Book* and *The Brown Book*; the latter's form he later called "boring and artificial".[12] He realized that he could---and *must*---continually detach, move, rearrange, amalgamate and reconfigure motifs and pieces of procedure and thought within one another without end, erasing and revising starting parts of thoughts once written down, shifting their force, revisiting themes and drawing out variations in a multitude of dimensions at differing scales, endlessly. He had faith in the unity of logic again, having surrendered the earlier ideals of simplicity.

The literary result is an "album", a landscape of shots of voices and variations, echoing and cancelling one another with modulations, self-confessions, re-phrasings, and artificial and natural snapshots of philosophical activity. It is dialecticity with precision and without loss, rigorous in its own way. It has been called an "intentional naïveté"[13], but we might think of it more specifically, as a way of thinking about what it is to think through *to* the simple, the unvarnished and natural, which is, at the same time, a rich field of unending depth and sophistication, a series of arguments about what is to count as simple, straightforward, obvious, or given. Wittgenstein

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[11] MS 111, p. 31 (1931).
[12] Engelmann (2013) quotes and analyzes this.
[13] Goldfarb (1983), p. 269.

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had thoroughly reflected on the very notion of logic, logical analysis, philosophy, itself. That reflection gets back, ultimately, to *Lebensformen*.[14]

What stimulated Wittgenstein to move to the fluidity of simples? I have a suggestion. During just this time—we are not sure when exactly, but it could have been any time from the summer of 1936 to the spring of 1937—he read Alan Turing's "On Computable Numbers: with an application to the *Entscheidungsproblem*" (1936/7).[15]

In that epochal paper, Turing *analyzed* our very notion of a *formal system*, the very notion of a "step" in a formal system, the very idea of "calculation in a logic". He did this by reducing the ideal of logic as a calculus to intuitive, picturesque terms, boiling it down to its most human, simple, elements: a set of commands, a piece of paper, a pencil, a bit of memory, routines and step-by-step movements. Dare I say it: a form of life. I think *this* fascinated Wittgenstein and turned him toward his mature philosophical method.

Turing's analysis—inspired, I believe, by Wittgenstein's lectures and dictations—is a logical must.[16] If one wishes to clarify what the significance and nature of such things as formalisms are *in general*, one had better not simply write down another formalism, or keep on

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[14] A later use of *Lebensformen* (probably after 1946), published in RPP I §630 makes this association with simplicity clear:

1298.«5» Statt des Unzerlegbaren, Speifischen, Undefinierbaren: die Tatsache, daß wir so und so handeln, z.B., gewisse Handlungen strafen, den Tatbestand so und so feststellen, Befehle geben, Berichte erstatten, Farben beschreiben, uns fuer die Gefuehle der Andern interessieren. Das hinzunehmende, gegebene - koennte man sagen - seien Tatsachen des Lebens. //seien Lebensformen.//

According to v. Waedelstaedt, the origins of this are in MS 133, p. 284 (7.11.1946), TS 229, p. 333 (1947) and MS 144, p. 102 (1949).

[15] In February 1937 AT wrote to his mother that he had "already" sent off a copy of the paper to Wittgenstein (Turing to his mother February 11, 1937 AMT/K/1/54 in the Turing Digital Archive http://www.turingarchive.org/browse.php/K/1/54.). Wittgenstein came second of the list of those whom he had contacted outside of King's colleagues, just after Littlewood.

The manuscript was given to Newman by Turing in April 1936, the very month that Church finished his own proof of the impossibility of an *Entscheidungsproblem* for logic. Newman did not read it until mid-May, thinking at first that it was too simpleminded. Only after reading Church's paper did he see the magnitude of what Turing had done, and it must have created a buzz in Cambridge circles. Turing submitted his paper 28 May 1936 to the journal of the London Mathematical Society.

By 28 August 1936 he had added an appendix showing that his definition of "computable" is extensionally exactly equivalent to Church's notion of "effectively calculable" (described by a formula in Church's lamda-calculus); cf. Hodges (1983/2012) Chapter 3. We cannot be sure of when Wittgenstein first saw or heard about the paper, though it seems not unlikely that by the summer of 1936 he had heard of the result, and perhaps even discussed it with Turing. We do know that he sought Turing out for a discussion of it and the bearing of undecidability results on the foundations of mathematics in the summer of 1937, when he returned from Norway with his *Urfassung* of the PI. See Floyd (2001b), Floyd and Putnam (2012).

[16] Cf. Floyd (forthcoming c).

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urging people to do so.[17]   Nor can one simply invoke the idea of a "metalevel", for that has no hope of analyzing the *general* idea. Instead—and in order to show what formalisms *cannot* do--- which is above all what Turing showed in "On Computable Numbers"—one has to do something different, get a general *analysis* of the very idea of a "systematic procedure" in the relevant Hilbertian sense. Here one cannot resort to formalized means, though whatever one does, it must be robust enough to handle *any* and *all possible* formalisms. The very idea of a "letter", "phrase" or "symbol" must be indefinitely schematized. And this is what Turing did.

Turing's analysis turns, in the final analysis, on an *informal* comparison between a human calculator using pencil and paper, following a series of determinate commands, and a mechanical process implemented by a machine.[18] This is why what he did, though utterly rigorous mathematically, is a piece of "applied philosophy", a "paradigm of philosophical analysis".[19] Turing framed a *language game,* probably indebted to Wittgenstein's lectures and dictations.[20] And in terms of this object of comparison he was able to resolve the *Entscheidungsproblem* for logic---what Ramsey had once called a "leading problem of mathematical logic" (cf. PI §124)--- in the negative.[21]

Moreover, his "diagonal" argument for this result was idiosyncratic, couched in terms of an *empty* command, something of the form, "Do what you do", something even more bare than a tautology, not relying on negation.[22] He had made a Wittgensteinian assault on the ideal of exactitude expressed in the *Entscheidungsproblem*: resolving it from *within* logic, *by means of* logic alone.

The notion of a decision procedure for logic as a whole ultimately falters on two things: an analysis of what a formal system *is*, and the construction of a rule that *cannot* be followed without supplementation of a context. Turing provided these.

Gödel himself described the *absoluteness* of Turing's analysis—its remaining the same, regardless of the formalism involved—as nothing short of a "miracle":

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[17] This, I think, is why W. said he would throw up his milkshake at the bar if Findlay went on talking about Carnap's merits; cf. Findlay (1985), p. 58. On Turing vs. Carnap cf. Floyd (2012b).

[18] Cf. Turing (1936/7) §1.

[19] Gandy (1988), p. 85; Davis (1982), p. 14.

[20] Floyd (forthcoming c).

[21] Ramsey's remark opens his (1930), p. 26. On "leading problems" see PI §124(d), an allusion to Ramsey's remark in his paper resolving a part of it.

[22] I analyze the structure of this command in Floyd (2012c, forthcoming c). Wittgenstein knew this; on which see footnote 103 below.

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> It seems to me that [the] importance [of Turing's analysis] is largely due to the fact that with this concept [computability] one has for the first time succeeded in giving an absolute definition of an interesting epistemological notion, i.e., one not depending on the formalism chosen.[23]

Turing established, in fact, the general applicability of Gödel's incompleteness theorems.[24]

Turing's paper *must* have struck Wittgenstein. He had been the first to frame the idea of a decision procedure for all of logic in 1913, in a letter to Russell: this was *his* problem.[25] The Hilbertian ideal expressed in the *Entscheidungsproblem* was in a sense the ideal of the *Tractatus*, even if Wittgenstein may be said to have realized some difficulties with that idea early on.[26] As I have said, Turing's paper as a whole, with its argumentation, has a Wittgensteinian air, and that it is very likely that Turing got the methodological idea of the analysis partly from Wittgenstein, to whom he already exposed as an undergraduate.[27] Here, turning to *Lebensformen*, I will suggest that Wittgenstein himself was so struck by Turing's analysis that it was likely to have been part of what solidified his mature conception of philosophical method, a form of response, a thinking through to the foundations of what would be required. It drew *Lebensform* into his philosophical repertoire.

Turing's "Universal Machine", with its demonstration of the universal fluidity of simples (the issue of *interface* between software, hardware, language and environment, human and machine, as we might say) would have given Wittgenstein faith in his own new ideal of fluidity, as well as the fundamental power of his uses of language-games. It could itself be mathematically and logically analyzed. It would hold up. An absolute, self-standing conception of logic could be coupled with a primary emphasis on the fact that "common sense" (scrutinized endlessly, of course) would be required to fashion routines and allow us to understand and implement logic. The dream of an overarching *calculus* of logic had to be surrendered: the

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[23] Cf. Gödel (1946), p. 150, discussed in JF (forthcoming c). Compare Sieg (2006), especially pp. 472ff and Kennedy (2014).

[24] In a note added in 1963 to a reprinting of his famous 1931 incompleteness paper ((1963/1986), p. 195), Gödel called Turing's analysis "a precise and unquestionably adequate definition of the general notion of formal system", allowing a "completely general version" of his theorems to be proved.

[25] McGuinness (2008), pp. 56-69; cf. Dreben and Floyd (1991), p. 32.

[26] The literature on whether or not the *Tractatus* was committed to a decision procedure for logic is longstanding, stemming from early debates between Fogelin, Geach, Soames and Sundholm (cf. Sundholm (1991), for example). I believe it was not; arguments were given in Dreben and Floyd (1991), Floyd (2001a), my reading made more precise in Rogers and Wehmeier (2012) which argues that full first order logic without identity is encoded. Weiss (2013, forthcoming), so far as I am concerned, at last satisfactorily analyzes the logic in the *Tractatus* as a predicative fragment of second order logic; in particular, he precisely measures the (very high) complexity of the definability of logical consequence in this situation.

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Entscheidungproblem was resolved in the negative. Logic and our ideals of precision could survive this, but only by transforming them.

Turing would eventually and repeatedly express his strong agreement with, and indebtedness to Wittgenstein, and on just these philosophical insights. He made explicit pleas for investigating "ordinary phraseology" in the development of logic.[28] The very last paragraph of the very last paper he wrote reads thus:

> These [limitative] results, and some other results of mathematical logic may be regarded
> as going some way towards a demonstration, within mathematics itself, of the
> inadequacy of 'reason' unsupported by common sense (1954, p. 23).

Lebensform in Wittgenstein becomes a kind of replacement for this version of the Cambridge notion of "common sense"; a rigorization of it suitable to Wittgenstein's mature conception of philosophy.[29]

In the summer of 1937 Wittgenstein discussed his evolving ideas with Turing, pursuing issues in the foundations of mathematics stretching back to the Greeks.[30] Immediately after this he returned to Norway, he composed during that autumn and subsequent year the first full length draft of the *Investigations* (the so-called *Fruhfassung* , or "Early Version"), a continuation of TS 220 with what would become TS 221 (with a Foreword in TS 225). He would submit this as a manuscript to the Cambridge University Press in 1938 as part of his bid to become Professor.

The second part of this manuscript, TS 221, was an application of the first part to logic and the foundations of mathematics. It was eventually published posthumously, as RFM I. Wittgenstein always envisioned his second book, until the end of his life, as having a particular structure, which TS 221 instantiated. As a matter of fact, it was the same structure Frege himself envisioned, in laying out the idea of a book *he* hoped to write on "logic".[31] In the first part, the nature of sense, thought, and meaning would be clarified. In the second part this conception would be applied to logic and the foundations of mathematics. That second part was a *constraint* on success, for Wittgenstein, as it has not always been for his readers. He bound *himself* to this, as a standard of rigor. (*The Big Typescript* (TS 213) takes this form also, of course.)

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[27] Floyd (forthcoming c).
[28] Cf Turing (1944/5) and my commentary on it Floyd (2013a).
[29] Floyd (forthcoming c).
[30] Floyd (2001b), Floyd and Putnam (2012). As these works explain, Alister Watson's (1938) paper was at issue.
[31] Frege, (between 1879 and 1891).

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The early version of the *Investigations* (1937-8, the so-called *Frühversion*) had just this structure. The reason its alternative continuation of the first 186 sections of the drafted *Investigations* (later published as RFM I) took the form it did, is, I think, because Wittgenstein had reflected on Turing's "On Computable Numbers" and discussed it with Turing just before he composed it. Those discussions, and Turing's analysis, stimulated him to believe that his final conception of simplicity as *fluid*, rather than merely *relative*, could be—must be—rigorously carried through, applied to logic and the foundations of mathematics. It was this that he continued to work out in conversation with Turing in the spring of 1939 in his lectures.[32] Such remained the envisioned structure: even in his latest version of the *Investigations* he inserted a remark saying that the kind of thing he had done could be applied to the foundations of mathematics.[33] TS 221, the *Frühfassung* itself, ends grappling with the metaphor of the chain. If I am right, that metaphor was again revisited with *Lebensform*, in Part II of the *Investigations*. But I anticipate.

I am arguing that while it occurs but five times in the *Investigations*, the timing of the initial occurrences of *Lebensform* in Wittgenstein's manuscripts is important: 1936/7. For the uses are embedded in a wider flow of thinking about the concepts of *form* and *life*, and, most importantly, the notion of the *logical*. In a broad sense, in Wittgenstein's philosophy the accent seems to approach more and more to *life* as time goes on. But logic, with its idea of *form,* is never left behind as part of the quarry. Wittgenstein still conceived himself to be pursuing the nature of the logical, conceived of what he is doing *as* logic, sees philosophy *as* logic, until the end.[34]

What *is* the nature of the logical? Logic alone, in reflection, can get at this. It's a "big" question—maybe the main question, if, as I think we should, we count logic as philosophy itself for Wittgenstein. It is a question he thinks and works through. One can surely even read the *Investigations*—including its uses of language-games and its stress on aspects—as addressed, throughout, to this question.[35] I think for this reason we do well to treat *form* first, and *life*

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[32] Cf. LFM, discussed in Floyd (forthcoming c).
[33] PI PPF xiv 372.
[34] OC *passim*; esp. at §§68,82,375,501.
[35] Here I agree with recent work of Kuusela (manuscript) and Gandon (forthcoming), who have developed the idea. This conviction shaped earlier work of mine; for the thread in philosophy of logic cf. Dreben and Floyd (1991); Floyd (1991, 2000, 2001b, 2005 (p. 81), forthcoming a, c).

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second, though life with words *is* also logic, so it is not that we are ordering topics in terms of their subject matter or importance *überhaupt*.[36]

By "logic" I shall mean a primordial notion, i.e., not something mathematically articulated, necessarily, nor, in the end, a technical area of philosophy, even if Wittgenstein is surely critically working through *these* ways of construing "logic" as he goes, and even though mathematics (including the mathematization of logic itself) presents—as Wittgenstein repeatedly says and notes—perhaps the best concrete examples for chewing on and displaying aspects of its nature directly and intuitively.

This is consistent with---perhaps even entailed by---the idea that "life form" as it may ordinarily be meant includes the idea of a "living system", both the idea of an individual and that of a species, and also habits, reproducible patterns, or characteristics of growth (and decay) in certain living forms, individuals, cities, cultures, planets, including perhaps the (evolving) patterned uses of words.[37] There is a great deal of confusion in ordinary usage here about our criteria for *life*, confusion it is important for philosophers to explore: "common sense" and "ordinary phraseology" are essential here, just as Turing said.[38] If---thanks partly to Turing---we are to face at some future point synthetic human life, the embedding of computational devices not only in our lives, but even in our bodies and human forms, we must analyze the notion—as well as the logical notion of *form*, now orbiting in the arena of the idea of a *computable process*.[39] Wittgenstein insists in the *Investigations* that the only way to understand the concept of *life* is *procedurally*, from where we now stand, not conceptually, in terms of *Merkmale*. Such prescient insistence it was, coming from an insightful engineer who always remained a philosopher of logic---like Turing---to the core.

In what follows I inflict the nature of the logical, of analysis, of the regressive method, upon Wittgenstein's own ordering of *Lebensform* in the *Investigations*, regarding the whole as

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[36] I suppose that the *Tractatus* is also concerned with life, not only with logic; cf. Tejedor (2015) for discussion of he origin of *Lebensform* in the concept of *form* in TLP.

[37] In his second motto in *Word and Object* (1960) Quine echoes Häckel's (now refuted) biological principle that "ontogeny recapitulates phylogeny" with a quote from James Greier Miller (a theorist of living systems and adherent of the "Gaia" hypothesis of the earth as a living thing): "ontology recapitulates philology". How far studies of child language-acquisition realize a version of the principle as one I won't answer here. Quine's suggestion—perhaps registering dimly a part of a strand in Wittgenstein—is that we fill in Augustine's picture of language, and the determinacy's of meaning, by attending to how we evolve and learn in entering as humans into a language and mastering a variety of technologies.

[38] Cf. Thompson (2008), Kerbe (2016).

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itself such an exercise. For me, "Part II" will always remain "Part II", though I am fully aware that it has been renamed "Philosophy of Psychology: A Fragment" in the new 2009 edition, and I concede to "PPF" referencing and section divisions below. This titling is a pity. Neither Turing nor Wittgenstein are given proper credit for being philosophers of logic, first and foremost. Philosophy of mind is an *application*, and far less fundamental to what they did. If what I say below has merit, we need to rethink this titling, exposing its prejudicial force, which has led more than one generation of interpreters away from the heart of Wittgenstein. *And* Turing, though that is another story).[40]

Frame

The five occurrences of *Lebensform* in the *Investigations* will organize the discussion that follows, divided into four parts to encompass our themes:

- PI §19 and §23 (where the *variety* and *innumerably open-ended, active* (imaginable, adaptational) uses of languages are stressed),
- PI §§240-242 (where mathematicians' lack of coming to blows, scaffolding, *Übereinstimmungen* and the character of logic are treated)
- PPF i 1ff (where modes of the complicated forms of life—hope, grief, handwriting—are invoked, along with the idea of *Lebensteppich*)
- PPF xi 345-362 (where certainty and imponderables are brought in to the discussion of seeing an aspect, and what has to be "accepted", "the given" is brought in, alongside the metaphor of *Lebensband,* and the mathematicians are revisited).

The idea is to embed these occurrences and clusters of thoughts in wider themes by means of the following sectioning:

1. Forms of Life are Linked, not Ranked (PI §§19, 23)
2. Natural History
3. Chains, Scaffolding, Calculation (PI §§240-242)
4. The Band of Life (*Band des Lebens*) (Part II, §i) and The Accepted, the Given, as Aspectual (Part II, §xi)
5. Finale: "Culture" and "Value"

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[39] Turing himself wrote on morphogenesis (1992), mathematically modeling the evolution of patterns in the growth of flowers and pinecones.

[40] Cf. Floyd (forthcoming c).

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1. *Forms are Linked, Not Ranked*

In Wittgenstein forms have to do with *logic*, and logic is construed *modally*, in terms of possibilities and necessities (as it was not in Frege and Russell).[41] One risks here being accused of inflating the term "form" into a "technical" or limited or even "metaphysical" one, when it is actually an everyday word of German, and used by Wittgenstein just so in the *Investigations*, e.g., for "shape" or "appearance".

Nevertheless, I think we are obligated to take the risk in conveying Wittgenstein's ambitions and specific self-conception.   For he is bold enough to want to try to wed or embed or imbricate, analogically speaking, the technical conception with the more everyday, "common sensical" or ordinary one: to weave them together in complex ways. I propose exploring Wittgenstein's uses of the metaphor of *chains* to specify one understanding of *Lebensformen*, and to elucidate in what sense(s) they are "to be accepted" as "that which is given" (PI II, xi, §345). A conjecture at the outset is that they are "to be accepted" in the same sense that logic is "to be accepted".   But how is that? Not so easy to say, Wittgenstein shows us.

As in Russell, as in Frege, "logic" in this primordial and encompassing sense *is* philosophy in Wittgenstein. As I shall use the term, "logic" has to do with something irreducible, but everyday: the everyday sense of back and forth of argumentation, reasoning, reflection on such, and dialectic: Yes and No, but also here and there, correcting, pointing out, qualifying, precisifying, organizing and amalgamating and sharpening and dulling concepts, and so on. Aspects of ordinary parlance, these. The *Investigations* is in this sense a book of, and an investigation of, logic—embedded in the philosophical tradition, critically, of course.

First and foremost Wittgenstein concerned himself with forms of human life in which dialogue and procedures are imbricated; as his work goes on, these become situated in a ubiquitous, evolving, indefinite backdrop of contingencies, necessities, procedures, habits and possibilities that becomes ever more thematically rich and complex. More and more is encompassed within the investigation of "logic"—as if Wittgenstein's relentless quest to embed and retain logic's ubiquity in logic did not end. The evolution of his philosophy was not neat, but it can be placed into this kind of frame. This is part of what I shall attempt to do here, with all the risks of distortion involved. I am going to take it that *Philosophical Investigations*, and

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[41] Shieh (manuscript) details this point.

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everything before and after it, are just as much remarks in and about logic as the *Tractatus* was. This will give us a chain of links to trace through.

This metaphor of the *chain* is a useful one for orienting ourselves with respect to Wittgenstein's attitude toward the notion of form. It is a very old one in logic, and an old one in Wittgenstein. It appeared in the *Prototractatus* and in the *Tractatus*, and was subsequently developed and explored throughout *Philosophical Remarks*, *The Big Typescript*, *Remarks on the Foundations of Mathematics* and elsewhere—as well as in the *Investigations*, where it is even used in the Preface to describe units of his mature style of writing.[42] Significantly, as I have said, it is with an exploration of *this* metaphor that TS 221, the second half of the *Frühfassung* of the *Investigations*, ends.[43] It is as if Wittgenstein hadn't quite worked that metaphor all the way through. For that, I would like to suggest, he would need *Lebensformen*.

As I shall put it on Wittgenstein's behalf, *forms are linked, not ranked*. Logic in the primordial sense I am using it includes reasoning according to groups of considerations or beliefs that we have "chained" together into a certain order or structure, whether in ordinary language or formalizing.

The simile of the chain may certainly be found in other writers, of course. It is as old as the hills. Perhaps as old as philosophy. It is all over Frege, bound up with his idea of a "gap-free" presentation of reasoning (chains *hold together*). In *Was Sind und Soll die Zahlen?* (1888/1996) Dedekind made of *Kette* a technical notion so as to formulate the whole idea of a recursive specification of infinite set, thereby analyzing the series of natural numbers as a "system".[44] The "most difficult" part of his task in probing this far into the foundations was, he later wrote, his learning to conceive the numbers, as he put it, "naïvely", as a successive, step-by-step created possibility of an infinite series, allowing a 1-1 but not onto mapping of the system

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[42] Refs needed;

[43] Wittgenstein (2001).

[44] So Dedekind (1888/1996), p. 768:
I regard the whole of arithmetic as a necessary, or at least natural, consequence of the simplest arithmetical act, that of counting, and counting itself is nothing other than the successive creation of the infinite series of positive integers in which each individual is defined by the one immediately preceding; the simplest act is to pass from an already-created individual to its successor that is to be newly created. The chain of these numbers already forms in itself an exceedingly useful instrument for the human mind; it presents an inexhaustible wealth of remarkable laws, which one obtains by introducing the four fundamental operations of arithmetic.

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into itself.[45] For in the end, he knew that our "natural" numbers, "old and familiar friends", must appear to us to be caught *in* his structured analysis.[46] Russell used the metaphor of the "chain" in the Preface to *Principles* to refer to "chains" of reasoning as they would be presented formally in *Principia*, to mathematicians.[47] Lewis Carroll, in his ironic book on Euclid, uses the metaphor too.[48] Even Schopenhauer did.[49] It is characteristic of Wittgenstein, thinking through the nature of the logical, to zero in on the metaphor.

Wittgenstein was quite aware that he wrote in a post-Aristotelian world, in which perception would not suffice for any automatic or natural sharing of form unaided by artificial forms of articulation. He lived during a time in which the form/content distinction itself was being radically recast as an ideal of the systematicity of judgment. An overarching systematicity of possibilities and/or actualities could no longer be accepted *a priori*, even as an ideal, as a collectivity of form, ordered into a given, ranked whole. This much incompleteness does tell us.

But what does this metaphor of the chain involve? How might it be updated? What does Wittgenstein *do* with it?

Forms are linked, not ranked, in chains. Chains are put together, arranged, and then hold fast, articulated. Chains may be linked to one another, side-by-side or end to end or interwoven as in a coat of mail. Chains may be folded and unfolded (the root of the idea of a conclusion being *implicit in*, unfolded from, the premises), their links substituted for one another indefinitely. Their links may be broken off, reassigned, moved, amalgamated, broken into further intermediate links, and so on. Some chains are made of paper, some of wool, some of iron. Some are linear, some are tree-like, some are themselves closed loops. In general, the "being with in a chain", like the notion of "being with in a *queue*", is something we do *with*, not *beside* or *to* others. Chains may be reproduced, copied in the manner of pictures: they are, or can be made, *Übersichtlicht*.[50]

For a trenchant analysis see Sieg and Schlimm (2005), Sieg (2013), p.46. Russell discussed Dedekind's notion at length in *The Principles of Mathem*atics (1903), so that Wittgenstein would have known it from there, at least. We do not know whether he ever read Dedekind directly.

[45] Dedekind (1890/1967).

[46] Dedekind 1888/1996, p. 791; cf. Floyd (2013b), at 1026.

[47] Cf. Russell (1903), Preface.

[48] Dodgson, *Euclid and his Modern Rivals*, p.47; cf. Appendix below.
[49] Schopenhauer (2010), I Preface, IX, 43, 49-53.
[50] Cf. Mühlhölzer (2010) and Floyd (2015).

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We are responsible for fashioning the links, even in the *Tractatus*'s philosophy. There it is simple objects that are said to hang together in the *Sachverhalte* as in a chain, and names are said to be linked in elementary sentences in a *Verkettung*.[51] Yet only in the later Wittgenstein does *how* we link, what we specifically *do in* linking, become a thoroughgoing, integrated part of logic, part of life itself, rather than (as in the *Tractatus),* something irrelevant to what is bound to show forth anyway, no matter what we do or say or try to picture or imagine or fashion or how we live.[52]

Early and late for Wittgenstein, *in* logic (philosophy) our linkings are part of reality, and our notion of reality, as such. They can be — must be —broken off, returned to, rearranged, and broken off again. Arguments are like music: something composed, fashioned, heard, voiced, and listened to, singly and in groups. Crucially what they do is to show us *possibilities* and *necessities*. Logic does not represent the world as it is. It composes it as it may and must be. As the song has it: forty-nine reasons, all in a line, all of them good ones (bound with the links of logic), all of them lies.[53]

We are *binders*. We bind ourselves to one another and to procedures. We band into groups, friendships, cities. We band together words, arguments, volumes, stories, lies, ponytails, tapestries, costumes, dress. We band into musical groups, peoples, fans. We band remarks and reminders. The regressive culmination of Wittgenstein's remarks on *Lebensformen* in the *Investigations* occur in Part II of the *Investigations*. Here the notion of a "carpet" or "tapestry" of life [*Lebensteppich*] (PPF i, 2) is developed into that of a *Band des Lebens* (PPF xi 362). This is translated by Hacker and Schulte as a "weave" of life[54], picking up on the connection with the image of *Lebensteppich*; Anscombe had "pattern in the weave of our lives", attempting to be a bit broader. But *Band* is broader still, and more pertinent to Wittgenstein's quarry.

Wittgenstein's image of a *Band des Lebens* is his answer to the whole idea of a book of life, perhaps even *this* volume's life, perhaps his own life's work in philosophy (in logic). It is a reworking of the ancient idea of chains and bindings in logic. It is an image of, for, in and by logic.

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[51] TLP 2.03, 4.22, quoted below in Appendix.
[52] Cf. Narboux (forthcoming) on showing forth anyway, discussed in JF (forthcoming a).
[53] "Forty-Nine Bye Byes", on Crosby, Stills and Nash Greatest Hits 2005.
[54] Stern (2004), p. 167 quotes RPP II §§624-9, which use the interesting notion of a "filagree".

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We are *binders*: of words and sentences, of procedures, of routines and activities, of ourselves with one with another, of thoughts and words and actions. We put elements together in arrangements that hold. We are capable of binding ourselves to procedures and routines. We may use a band as a reminder. Chains and bindings can be broken, or broken off, or amalgamated, or simplified, or rearranged, shuffled, entwined, according to purpose—or, in fact, merely aimlessly, emptily, or else decoratively. In Wittgenstein's mature philosophy, these bindings must be fashioned without a base-level of fixed simplicity, even though every binding entangles us with simples. In the *Investigations* at PPF xi this idea of a *Band des Lebens* works out the whole idea of *pretense*, of lies, of what logic makes us capable of, and how. This is no accident. Wittgenstein is grasping at getting down to the very idea of dealing with possibilities and necessities of human actions with truth as such.

The philosophical self is not an object, nor even an ideal, a well-behaved limit of criterial or grammatical procedures. It is not to be conceived of as either continuous or "gappy"—as if something might be completed by cuts and collections, like the continuum was by Dedekind.[55] Instead, it is *bound* and *arranged*: bound by necessities and possibilities, including those of writing, those of a human body and bodies, of traditions and daily routines and voices—if not by fate. It is bound by its own bindings (of words and thoughts). It is held together by no more and no less. We should read life in language as we should read some books: aloud.

In the final culmination of *Lebensform* in the *Investigations*, Wittgenstein says that *Lebensform* is what "is to be accepted", is the "given". Logic is indeed given, but only in a very special sense. Our concept of life is given, but only in a very special sense. Life is not simply an observed *feeling*, as if an object of Russellian acquaintance, or something projected, aesthetically enlivened.[56] It is *there*, in reality, in possibilities and in necessities, in "forms". But it is not

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[55] Laugier (2005), p. 68 pointed me toward this passage at Z §648:
> One language-game analogous to a fragment of another. One space projected into a
> limited extent of another. A "gappy" ("*löchriger*") space. (For "inner and outer".)

I translate "„*löchriger*‟" differently than they way Anscombe does. The quotation marks clearly signal an allusion to someone else's use of the word. My sense is that Wittgenstein is critically alluding to the picture of using Dedekind cuts to fill in "gaps", as treated in Russell (1919). Dedekind himself strikes me as more careful with the image, especially in relation to Euclid (see (1888/1996), Preface to the 1st ed., p.793; compare (1872/1996) §3, p. 771 and Floyd (2013b) for discussion). In Wittgenstein's work on real numbers, he *always* attacks this picture of gaps, on which see Floyd and Mühlhölzer (manuscript). He is considering transposing his way of thinking about the foundations of mathematics into the problem of inner and outer here. Note that a chain *has* no inside, except when it is folded up, hidden through its arrangement.

[56] As Kant said in the *Critique of the Power of Judgement* (1790/2000) at 5:204, 277,

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simply the result of a long abduction, or stringing together of similarities. Nor are forms are
incorrigibly given constituents.

Instead, Wittgenstein holds, we must work to become *acquainted* with our concept of life,
to see *in* it its forms and necessities and possibilities, to become *acquainted* with simplicities,
and thereby with the concept of simplicity itself. This is what we do in logic, in philosophy, in
life itself. This notion of acquaintance is an everyday notion, woven into the notions of aspect
and of familiarity; it is nothing less than the sense in which, by looking, listening, probing,
discussing and responding, we become *acquainted* (with a person, an emotion, a proof, a
language, a life).[57] It is Wittgenstein's altogether worthy replacement for Russell's notions of
acquaintance and aspect.

Wittgenstein's philosophy thoroughly de-psychologizes the whole notion of life, drawing it
into the very nature and foundations of logic. The criteria we might think we can deploy to
distinguish animate from inanimate objects don't hold up (movement, for example, as in the
Investigation's wriggling fly; or *Gestalt*, or our willingness to use or not use the concept of
"automaton").[58] We are *acquainted* with life: in Wittgenstein's aspectual, comparative,
ordering, procedural, actionable way. But this is a complex and delicate matter, a matter of give
and take, warp and weave, listening and speaking, making gestures and remaining still.

Wittgenstein pursues a one-leveled, multi-aspectual view of form, not ascending to a meta-
level or supposing that form may be reduced, Hilbert-style, to strings or signs or an axiomatic
formulation of a theory, or lodged, Hardy-style, in actual, albeit purely metaphysical conceptual
structures (propositions) intrinsically true or false. This is a tough and idiosyncratic road: useful
analogically, but cutting very much against the usual grain of discussions of intension and
extension, concept and object, in traditional logic.

Yet if one treats logic as "merely formal" in Hilbert's way, or the "formalist" way that
Frege was concerned to attack, then one is *forced* to say that there must be "metalogic",
something "outside" the formalism.[59] Wittgenstein always attacked this idea.

The huge shift from Aristotle's notion of form is that in Wittgenstein forms do not
ultimately have to correspond to any fixed tree-like taxonomic structure or ordered system of life
forms, essences, substances, or concepts, surely not one that is given or assumed ahead of time *a*

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[57] For an extended treatment of this replacement see Floyd (forthcoming a).
[58] PI refs

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priori (except relatively, and revisably, dynamically, as with chains or chains of chains). No such
hierarchical and grounded structure of forms is revealed in chains of reasoning. We cannot any
longer regard, e.g., particular syllogisms as something revealing life-form essences and their
(intensional and extensional) relations. In Wittgenstein a logical train of thought or an analysis
may compel us, but it need not tell us, either *that* or *why*—again, unlike in Aristotle. We begin
with such a chain, and we end with it. Logic does not represent: in it we propose, bind, and
dispose. It reveals itself *in* that, and that alone.

Forms—logical and living ones—are linked, not ranked. In Wittgenstein forms have to do
with procedures, routines, possibilities and necessities thereof---these are so ubiquitous. And so
he self-inscribes and self-thematizes the forms of his own procedures in the *Investigations*. His
remarks are deeply self-embedded in if-thinking, in *possibilities* of procedures or phenomena, in
loops and pieces of proposed and explored chains of argumentation. The same might be said of
the *Tractatus*, though there the idioms are less modular, because Wittgenstein's ideal of
simplicity is one of groundedness: as he later put it---critically---he had composed a "calculus of
indefinables".[60] Still, even there his philosophy of logic (his philosophy) is modal through and
through, based on a fundamental reworking of the whole idea of chaining in logic, drenched
with, and intended to illuminate in all its glory, the fundamentals of logic itself—by way of
logic.

In later Wittgenstein the modes (in the *Investigations* "modifications of this complicated
form of life" (PPF i 1)), are explicitly thematized as *aspects*, are contextualized and emerge from
structure and comparison, being treated occasionally compositionally, bottom-up. Wittgenstein is
inclined more and more, as time goes on, to stress the varieties of ways aspects emerge and strike
us, and the variety of our own activities in binding up chains of thoughts. He explores the
differences here between our capacity as binders and those of other life forms (dogs and rabbits),
but what is at stake here are *our* bindings, our concepts ("natural history" in *Wittgenstein*'s
sense). *Aspect* is a *logical*, not a psychological notion.

The metaphor of a chain naturally connects, then, not only to "form" but to "life". The
great "chain" of Being is no longer, as in Lovejoy, an ordered, gap-free taxonomic structure in
which all possibilities are eventually realized. It is a series of processes and fluid though

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[59] Even Schopenhauer falls into talk about "metalogical truths" of logic (2010), p. 56.
[60] Cf. MS 111, p. 31.

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structured events of formations of formations of (and in) life, necessities embedded in a world of contingencies, in what we can do and do and fashion, and what follows from that about possibilities for other forms. Such chain-procedures or possibilities, which we follow and put together and share, afford opportunities for exercises in characterization, analysis, and simplicity that are themselves designed to characterize, analyze and simplify the notions of characterization, analysis, and simplification. In the later, mature Wittgenstein chains have no absolute end or absolute beginning, in terms of a fixed place for measuring: there remains *in general* only the schematic "it begins" and "it ends", the possibility of binding (with words, thoughts, actions, streets, gestures, procedures…) *as such.*[61]

There is neither an absolute, nor a merely relative notion of simplicity here, and no doctrine at all of *un*definability. There is simply the ubiquity of the capacity for the binding. This can sound like an evolutionary picture in Darwin's sense, and the affinities between Wittgenstein and the pragmatists, for whom Darwin was a central figure, have often been noted.[62] While he would not have denied that logic has evolved from somewhere, historically, Wittgenstein would also have insisted—every bit as much as Quine, in his anti-reductionist moments—that we *make* history, as actors within it, and especially by talking with one another.[63]

The words, on this picture, with their evolutions of patternings and usages, are part of our dynamic reality. They do not, of course, *constitute* it alone: Wittgenstein is no linguistic idealist. Furthermore, unlike Hegel, Wittgenstein does not suppose that there is any particular end of history to be expected: Why would he? If history has ended, if an ultimate analysis of logical analysis itself (both *logos* and *logistike*) has been gotten to, then this has happened by embedding it in everyday life, ubiquitously. Then it is with a schematic way of thinking that we must end, in the final analysis. The *Investigations* gives us this, an overall moment of orientation, an adaptation of Turing's comparison between our calculational activities and machines that work us and work on their own commands, software and hardware, endlessly, without limits of time, but incorporating all the same the dynamic notion of a step-by-step procedure or routine. Life becomes ubiquitously bound: to transcription, to recording, to voicing, to automaticity.

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[61] The resonance with Turing's work on morphogenesis (1992) is striking.
[62] Cf. e.g., Misak (2016).
[63] Cf. WVC p. 34, n. 1.

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We live in Turing's world. Wittgenstein, his teacher, trained as an engineer, philosopher to the core, appreciated that we would.

2. *Natural History*

But now I hear an objection: does Wittgenstein not *replace* the notion of logic with that of grammar? "Grammar" is most often applied to *particular* expressions in the *Investigations*, usually words, verbs, or brief phrases; a grammatical "investigation" is also connected with the *kind* of object an object is, with *possibilities* of phenomena. It is evinced in an exploration of our criteria, which show us the place of a concept: possibilities of use, not the actualities of particular objects or uses *per se*. In Wittgenstein, grammatical explorations are, in my broad sense, logical: they reveal *aspects* of thoughts, words, sentences, states of affairs, ourselves, and so on. For they open up fields of possibility and necessity that are made, in logic, alive to us. On my view *aspects* are not something "subjective": they are there to be seen, even if they require our activities to be drawn out, or revealed, or composed, *formed*.[64] *If* there is ever anything going on analogous to therapy in the later Wittgenstein, it involves not simply liberation through talking *per se*, but this only in seeing and taking in and living with possibilities and necessities.

With "grammar" Wittgenstein is making a particular sort of move in relation to the tradition. He is insisting, not terribly novelly, that grammatical explorations are part of logic: Aristotle said as much. But he is not *replacing* logic with "grammatical investigations" only, for their own sake. This idea is—unfortunately— encouraged by the finally published form of the *Investigations* (which lacks TS 221, the application to logic and the foundations of mathematics) lending to Wittgenstein's inheritance too great a focus on language as such.

After the mathematicization of logic in a *language* post-Peirce and Frege, mathematicians such as Hardy and philosophers such as Carnap stressed that ordinary parlance is vague, mushy, unrigorous, a seat of endless and fruitless argumentation. Hardy and Littlewood called this "gas", and "provisional nonsense".[65] Carnap recommended the replacement of ordinary phraseology with the construction of "formal", axiomatic systems. The *Investigations* is insisting, very differently, on getting at the "gas": the analogies, the metaphors, the nonsense, and so on.

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[64] Cf. Floyd (forthcoming a).

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Wittgenstein wanted to show mathematicians, logicians and philosophers that how we use words in everyday life (including mathematics) is inseparable and not to be dismissed from foundational talk. A philosophical investigation would not be merely *decorative*, it would be an essential part of the work.[66] Wittgenstein was not the only logician to think this way after Gödel: Gödel *himself* did, *himself* thought that to get the foundations right we had to go *outside* of mathematics, to philosophy, to everyday life.[67] Turing thought this way too, explicitly indebted to Wittgenstein's teaching.

This is why much of Wittgenstein's method turns on exploring what we are actually inclined to say about what we might possibly say or do (or not do). But the quarry is (the nature of) logic, not word-use for or particular concepts, or chatting *per se*, even if *these too* would receive re-thinking in light of philosophy. In the *Investigations*, the distinction between "prose" and "proof" becomes permeable, though still able to be drawn locally, as he comes to see the ubiquity and complexity and fluidity of reflection, both on and in "prose" and "proof".[68] This is very clear in TS 221, but, as we have said, this part was excised on the way into the second major draft of the *Investigations*, the *Bearbeitete Frühfassung* (TS 239).[69]

Wittgenstein counts "chatting" [*plauschen*] explicitly among those most pervasive elements of our "natural history" in the *Investigations* (§25): this is the culmination of the first occurrences of *Lebensform* (in §§19, 23), which I shall not bother quoting here. These take off from the metaphor of the city as an evolving organized whole, its "downtown" heart centered on ancient parts rather than superhighways. The *Investigations* is not just chatting, however. It comprises *argumentation*, a lot of it — something I have found those who write about Wittgenstein's philosophical "methods" as if they are primarily directed at the nature of language and meaning too often underemphasizing.

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[65] Cf. the Preface to Littlewood (1926) and Hardy (1929), p. 18, discussed by Wittgenstein and Turing in Lecture I of LFM. I treat these metaphors at length in Floyd (forthcoming c).

[66] Cf. Lecture I of LFM, Floyd (forthcoming c).

[67] Cf. Floyd and Kanamori (2016) on Gödel's (1942-3) *philosophical* reactions to Wittgenstein and Russell.

[68] Cf. Floyd (2001b). Kienzler and Grève (2016) note the permeability point, but ground their interpretation of Wittgenstein's 1937 remarks on Gödel in the evolution of Wittgenstein's thought about language *per se* (cf. pp. 85ff), leaving philosophy of logic out of the picture. They attribute philosophical views to Gödel that, I believe, are in error (cf. Kanamori and Floyd (2016)) and, finally, misdescribe the "situation" in "ordinary mathematics" at the time.

[69] See Wittgenstein (2001) for these manuscripts.

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There is a difference between chatting, requesting, insisting, and so on, and arguing and genuinely reflecting on *that*. Part of Wittgenstein's philosophical aim is to teach us to characterize, negotiate, recognize, and utilize in our own ways these complexes of differences rightly. Philosophy turns on nothing more, and nothing less, than developing an ear, a nose, and a pair of eyes for the difference between chatting and arguing, questioning and supporting, qualifying and stating, negating, and misunderstanding. Understanding requires *work*.

By "natural history" Wittgenstein does not mean, paradigmatically anyway, the sort of thing that could be excavated out of a book about evolution that has diagrams or generic statements about the lives of animals, e.g., a naturalist's guide to the rockies. The best instance for what he is getting at here is, instead, the Natural History Museum in Vienna, a place he surely knew. There the point is the *presentation* and *arrangement* of natural forms into patterns that are beautiful, instructive, painterly, designed, formed, engendering conversation pieces.

The whole of *this* sort of "natural history" is an intended counterpoint to the Art Historical museum across the street. One of the great quarries of the *Investigations* is the issue of where artificiality and improvisation begin, and habit and convention end. This is part and parcel of his effort to characterize the nature of the logical. He was perfectly aware that his hero Frege would have discounted many of his reflections as pertaining only to the origins or psychological accompaniments of thought. He is insisting they are logical, through and through.

Regressive Method

A certain dialectic or tension between the way a proof is presented, and its *raison d'être*, is very ancient, a living character of the axiomatic tradition in mathematics as well as the analytic tradition in philosophy. Book I of Euclid's *Elements* serves to illustrate it. *Elements* Book I ends at Proposition 47, the Pythagorean Theorem. That is, in a certain sense, the *raison d'être* or goal of Book I. But the presentation, as so often in mathematics, is written *backwards*. The method of presentation is: *Given* this, you can get the theorem. *Given* that, you can get this. And so on.

Proofs, analyses, regressions, reductions to givens, are written *backwards*. So Hobbes read Book I of Euclid's *Elements*, if we can believe Aubrey's *Lives*.[70] There is a systematic

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[70] Stillwell (2010), 2.1, p. 18 quotes the passage; cf. Appendix.

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backwardness to be taken in. The "givens" are the beginning and the end, the interlinked chain as a whole, and also the particular linkings. *Necessity* appears, until it ends somewhere (*not* in self-evident "givens" or "aspects of acquaintance" in Russell's sense, according to Wittgenstein). *Sufficiency* appears to work frontward: *Given* this, you can get that; *Given* that, you can get the next, and so on. But we use this, in fact, to stich our way *back* to the "givens" or simples.

Reading forwards, the uninitiated reader may be told once, at the outset, "This is where we are headed: here is the goal of the journey". She may be reminded of this from time to time. But it is the journey, not the destination that counts. We mostly do not see, moving through as readers, where we are. The end is not yet chained, bound, in a series of procedures we can take in. The connection to the end may appear to lie somewhere underground in hidden machinery.[71] Maybe we will never see the whole as a chain. Maybe a piece of machinery, to which we offload, can help us to take it all in, archive and arrange it all, to see, take in, and form the necessities and possibilities in another way. Information requires formation, after all.

Of course, these images of "forwards" and "backwards" are insufficient, ultimately metaphorical. A chain has no such directions or orientation (unlike, say, an arrow or a pointing finger). The *Elements* Book I the aim is the composition and activity and arrangements, the chains of aspects themselves. The important thing the possibility of *this*. By means of it, the Pythagorean Theorem is not merely a statement or headline. It is transformed into a living face, something with colorful aspects. (Wittgenstein writes of the *discovery* of aspects, and the *invention* of techniques; he also writes much about Euclid. [72])

In "Declining Decline" Cavell suggested that if there is a philosophy of culture in Wittgenstein, it inheres nowhere else than in the very structure of the *Investigations* itself.[73] We shall return below, in the final section, to the issues of "culture" and "value" in the *Investigations*. For now I will note that Cavell's suggestion fits beautifully with a picture of the book as belonging to logic in the ancient sense: to philosophy, demanding as it does the representation of *logos* itself. From this panoramic point of view, the *Investigation*'s way of thinking must be self-standing and alone competent to judge its own workings, as well as fluid and ubiquitous, embeddable everywhere: an endlessly interesting, endlessly articulable way of

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[71] As in RFM I §106. App. II §3; cf. Floyd (2010, 2012).
[72] Cf. Floyd (1995, 2000, forthcoming) for discussion of Wittgenstein and Euclid, as well as BT pp. 105ff, 325-334.
[73] Cavell (2013).

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aiming to pursue---and also learn how to break off from---certain kinds of perfection, ordering, in thinking.

Cavell adduces the idea of "voice" here, looking at the literal forms of the *Investigations*.[74] Sandra Laugier has spelled this out into a compelling vision of "forms of life" as essentially structures of possible voicings.[75] I would only add to this the significance of Wittgenstein's *orchestration* of voice: *exemplary arrangements* and *linkings* of *forms* or *possibilities* of voice; compositions as opposed simply to chats, although chatting there is and---one would like to say---so it has and ever must be. Indeed: many of the snapshots and chains of voicings recorded in the *Investigations* no doubt derived, autobiographically, from the author's own interlocutories with his students and himself.

In every period of Wittgenstein's thought logic is *schematic*, though in the *Investigations* notably without the special logical vocabulary of *schemata*. That there is no special logical vocabulary in the *Investigations* is necessary to Wittgenstein's aim, and intrinsic to his idea of what logic is. It is mandated (as I have already said) by the task of logically analyzing logical analysis itself.

There is a dim but distinct echo here of certain *Tractarian* ideas, of course, now subsumed into the mature way of thinking: there is no special subject matter of logic (e.g., the "logical constants"—recall the *Tractatus Grundgedanke*). Logical words offer punctuation for possible thoughts, not reference to the constituents of senses.[76] What there are are chains of interlockings of *possible* thought-trains, and logic's conception of thought is itself a conception of expressions of agreement and disagreement with possibilities. Most of all, the key idea: *form is the possibility of structure* (TLP 2.032).[77]

As reflection and argumentation and chatting are pursued, they gradually assume the dress of further articulation. Such belongs to our "natural history". As we know, "logic" is eventually explored in the *Investigations* in relation to the imposition and concocting of possible procedures

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[74] The idea of a "polyphonic" structure of the *Investigations* has also been developed, to different interpretive ends, by Pichler (2004) and Stern (2004).

[75] Laugier (2015).

[76] Cf. Schopenhauer (2010), p. 53: "It may be said that logic is to rational thought as the figured bass [*basso continuo*] is to music". The idea is that the notation is important, not for performance, but for composition and criticism, allowing for "accidentals" and transpositions of key to be noted and enforced.

[77] See Shieh (forthcoming, manuscript) for an analysis that stresses the importance of modality in the TLP; cf. Dreben and Floyd (1991) for the same.

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or routines, and, more specifically, "calculi".[78] Such was logic's own history of formalization, our natural history with it.

This, we recall, so impressed the young Wittgenstein that he regarded modern mathematical logic as relating to its forebears in formal logic in the way modern chemistry does to alchemy.[79] That chemical metaphor is rejected after 1929, along with the *Tractatus's* particular ideal of simplicity as undefinable. It was replaced 1929-1936 with a notion of *relative* simplicity. But this was still a conception that dragged with it the shells of the *Tractatus* picture of analysis. Wittgenstein's mature ideal of simplicity, his getting to "the right method" was not really come to terms with until the spring of 1937, as I have said.   Then, seeing the negative resolution of the *Entscheidungsproblem*, he understood that there *are* no "leading problems" of logic.[80] All (and so none) of its problems are leading, for all of them lead and follow, are already before our eyes, are ubiquitous, are everyday.

3.   *Chains, Scaffolding, Calculation*

In a quite ordinary sense the notions of "juxtaposition" and "concatenation"— "chaining"—are ancient and fundamental to the primordial idea of logic I have characterized above. Certainly such phenomena of chaining are strewn across the history of mathematics, of logic, of alphabets and writing.   Philosophers recognized this long ago, beginning at least with Plato, who worried about the suppression of human voice in the face of writing.

The Babylonians appear initially to have put "7" next to "5" in *juxtaposition* to "add" and get twelve, a kind of fusion of constituents in which order is irrelevant (twelve sheep are "made up of" seven and another five). That was surely a procedure. Yet getting to *concatenation* ("chaining" or "stringing"), essential for getting in turn to *routinization* and iterable *calculation*

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[78] Alois Pichler draws on a deep understanding of the Nachlass when he emphasizes (2016) that the "calculus" and "human" conceptions of language use vied with one another in a complicated, interwoven manner throughout Wittgenstein's life, one sometimes ascendant, then the other, neither wholly supporting or occluding the other. I am proposing that "forms" partake of both aspects.   This reading is at odds with a straight-line progressivist interpretation on which Wittgenstein replaces a "calculus" conception of meaning derived from Frege, Russell and the *Tractatus* with an anthropological or ethnological conception, of which an open-ended, interlocking series of language-games, conceived as constituting "forms of life" or "cultures", are the expression (cf. Hacker (2015)). I regard my reading as a supplement to Engelmann (2013), where the dangers of a purely anthropological method are explicitly stressed.

[79] Wittgenstein (1913).

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was a large step from here. They were able to get to it (after some centuries) with their base 60 notation, thereby folding in what belongs in general to writing and ordering in the sense fundamental for any civilization, logical proceduring, or language. Concatenation draws in, *is*, ordering: perhaps first a unidirectional linear order of symbols, and then, later on, other forms of order. The decimal notation, and "+", came much later on, as did later still the more and more general combinatorial issues (of partitions of numbers, of possible arrangements of sheep and officers of different ranks, of a system of recursion equations, of the ordered pair, and so on).[81]

It is a remarkable fact that the regressive method itself only rendered a clear notion of *set* (unordered elements put together, *ensembles*, as the French have it), thereby reintroducing the ancient idea of static joining, so late in the game---in fact during Wittgenstein's early lifetime. Here is a "Given" *gotten to* through the regressive method, then capable of synthesizing an enormous playing field for mathematics in set theory. It was not one Wittgenstein particularly liked playing in, admittedly. But we can see why. For it obliterates, in its very nature, the aspectual, chain-like level of conceiving *procedures*, *putting together in orderings*. Thus it obliterates Wittgenstein's central *data* of logic, just as it had for Frege before him. It also invites in by the back door the whiff of both absolute and relative conceptions of simplicity, a one-sided, undynamic picture, in which forms, once again, are ranked but not (in theory) linked.[82]

Once *concatenation* is in place, eventually notions of *substitution* and *occurrences of a sign* require hard thinking through in logic. As a matter of fact, these were the very issues with which Wittgenstein and Russell strugged in 1913.[83] Substitution was only formulated as a separate principle of logic in C.I. Lewis (1918), and first characterized arithmetically in Gödel

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[80] Cf. PI §124, a reference to the *Entscheidungsproblem* as described in Ramsey in his (1930), which provided a very significant (positive) resolution of one kind of case. See MS 110, 189 (1930), BT pp. 417ff.; MS 115, 71 (1933).

[81] As Otto Neugebauer, the great historian of ancient mathematics with whom Wittgenstein shared a pencil while in Monte Cassino (cf. Swerdlow (1993)), once wrote (1935/1937, vol.3, p.80):

What is often overlooked and cannot be sufficiently emphasized is the terrible difficulty and slowness of the development of the very simplest fundamental mathematical concepts, first of all of a genuine computational technique.

For Wittgenstein on Euler's thirty-six officers problem, and the bridges of Königsberg, see Floyd (2012), p. 240. (As Wittgenstein wrote (PI §23, second occurrence of *Lebensform*), we get a "*rough*" picture of the varieties of uses of language from the history of mathematics. For some suggestions of contemporary issues illuminated by these themes, see Floyd (2016).)

[82] On Wittgenstein's sometimes crabby, but not utterly useless remarks on set theory, see Floyd and Mühlhölzer (manuscript).

[83] Cf. Russell (1913/1984). In Kanamori and Floyd (2016) it is shown that Gödel was well aware of these debates about truth and judgment, siding at a certain point---the theory of infinite order---with Russell against Wittgenstein.

(1931)—crucial preliminaries to Turing's "On Computable Numbers" (1936/7).[84] Each of these "chainings" instituted new "forms of life", and created and grew newfangled calculative behaviors on the part of humans, at the individual and the civilizational, cultural levels. As Wittgenstein argues, the notion of "following a rule" involves customs, institutions, *Praxis*. These are at *heart* routines that are to be shareable---just as Turing's "machines", with their commands are. (As Mühlhölzer has shown, in calling a proof *Übersichtbar*, or *Übersichtlich*, Wittgenstein meant, quite literally, "can be copied, in the manner of a picture": i.e., can be shared, communicated, reproduced, routinized.[85])

In Wittgenstein's way of conceiving logic, arguments are not merely "sets of sentences", but specific structures that are put together, "chains", offered in some (living) context or other, whether with words, diagrams, letters, or a combination of these. As parametrizations of thought-procedures, held together with "links" alone, they put together fields of possibility and necessity, and invite and draw out other possible constructions of related structures. If this follows from that, can it follow from those? How does a recursive proof differ from, or transform itself in a particular case, into a computation? What if I interpose a further step in this argument, or add such and such a premise? What if I qualify or shift my understanding of a premise, or a term? What if I contest the interest or point of this particular chain? What if I remove part of it, substitute another, and take the removed part elsewhere?[86]

Such a chaining (a possible procedure for thought, in certain cases regarded as a possible construction or computation or algorithm) draws out *aspects* of thoughts. These aspects are there to be seen and proceeded on the basis of: they are not merely "subjective", even if they are

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[84] The attribution to Lewis, who himself goes through much history along the way, may not be right. Sanford Shieh has suggested to me that Lewis may well have derived the idea of it from Royce, or, one might add, it may have come that way through Peirce. The use of "the method of substitution" in a broader sense is of course mentioned in the TLP (6.24), and has a long history in mathematics.

[85] Mühlhölzer (2006, 2010); cf. Floyd (2015) for discussion.

[86] I take it that Wittgenstein is revising Hilbert's idea of logic as part of the *Fachwerk* of a theory (cf. Sieg (2013) p. 24n for historical contextualization and discussion) by making it dynamic in our varieties of its uses. Hilbert had written to Frege (29 December 1899, cf. Frege (1980), p. 42 where (wrongly) "scaffolding" is used instead of *Fachwerk*) that "Any theory is evidently only a *Fachwerk* of concepts or a schema of concepts together with their necessary relations to each other and the basic elements can be thought in arbitrary ways"---something that evidently did not move Frege. The image of *Fachwerk*, however picturesque as a memento of Göttingen, is static, structural, holding a house together, decorative: Hilbert wanted an image of complete organization of a theory by means of its logical analysis and axiomitization. Wittgenstein counters with the idea of logic in *motion*: scaffolding that plays no structural role, and can be modularized, taken down and put back up again as needed.

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response-dependent in a number of senses.[87] They are brought out through nodes or parameters in arguments, that may or must sometimes be shuffled and reconfigured (see Appendix). They allow for "intermediate" links among cases through comparison, substitution or rearrangement of parts (Cf. PI §§122,161).

Logic draws aspects out of characterizations, it characterizes characterization itself, and discusses and reflects on this (i.e., on logic itself). "Same" and "different" are fundamental notions here, as are the notions of a "character" and "charactery" and order, handwriting with orientations of letters, specific arrangements, the notion of hanging together as a whole, while being step-by-step configured, link- or band-wise, in a strong, specific, but transportable structure that is contingent, as a matter of fact, but gives rise to step-by-step necessities, once it is configured.[88] Wittgenstein works hard on orientations of letters, even mentioning handwriting as an instance of *Lebensform* in the *Investigations* at PPF i 1.

Turing's analysis is impervious to what the specific signs might be in a particular computational routine. But a Turing machine has *two* faces: one static (a set of quintuples), one dynamic (the moving action of a machine given its commands). Turing has perfectly interwoven, in general, the interstices between juxtaposition and concatenation, without confusing them, has kept in play the dynamic *and* the static, the complex *and* the simple, placing them all into one self-standing space, sitting within our world of "ordinary phraseology". For the list of quintuples that define one of his "machines" are a *set* of commands that may be listed in any order (the concatenated ordering *within* each command is taken for granted). But what such a "machine" *does* is to concatenate, dynamically, procedures, moving forward.

The problem responded to is as ancient as Socrates's dream in the *Theaetetus*.[89] It is thematized as well Aristotle's mocking of the atomists in *Metaphysics*, who "lazily neglected" to face the issue of *movement* in accounting for the simples, thinking that the difference between AN and NA, or H and

$$\sqcup$$

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[87] Here I part ways with an aestheticizing and/or phenomenological sense of "aspects" as primarily experiential or "subjective" (cf. Baz (2000, 2010), Mulhall (1990, 2010)). I think aspect spans action and passion, response and imposition and perception---even in aesthetics, for it is connected to procedures and chains of patterning and simplicity, a journey. Cf. Floyd (forthcoming b).

[88] Cf. Floyd (forthcoming a).

[89] Cf. Parikh and Renero (forthcoming).

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could be accounted for in their terms alone.[90]

The logical notions of "concept" and "object" sit rather uncomfortably with this aspect-thinking, as neither extensional nor intensional ways of construing concepts fit directly into its way of articulating logic. Wittgenstein *has* to stress the importance of inventing "intermediate links". Procedures, with our modal and dynamic ways of visualizing, applying and imagining them, are a more natural fit with aspects.

Logic as Wittgenstein conceived it has to do, I am arguing, with investigations of *possible* juxtapositions, orderings, concatenations (chains), iterable procedures, rules and rule-following. This is why the notion of *aspect*, as opposed to *concept*, is primary for him. Concepts, rules for the uses of words, and so on are understood in terms of aspects, not the other way around; the family resemblance idea is *necessary* for Wittgenstein, not simply a picturesque idea of generality or historicity inheritance from Goethe and Spengler: he has *logicized* morphology, morphologized logicism. The idea of family resemblance is logically *robust*: like that of a chain or interwoven thread, as Wittgenstein says. It is mode-like, *modular*, in its very essence.

Felix Mühlholzer has remarked, in discussing Wittgenstein's later philosophy of mathematics, that it has a "dark" side: Wittgenstein rarely if ever discusses axiomatic method. That's right, but there's a reason for this. Like Turing—and well before he met Turing, when he wrote the *Tractatus*—Wittgenstein did not reduce logic to the axiomatic method. Instead, he reduced that method itself to the terms of more general and fundamental ideas: rule formulation, catenation, iteration, substitution, and calculation: procedures of calculation that begin somewhere, and may be carried on and articulated indefinitely. His struggles in his Cambridge lectures during the early 1930s zeroed in on how satisfactorily to formulate the very idea of definition by *recursion*, and, especially, the issue of *how to get the starting point right*---and, of course, how to get it wrong (cf. BT passages in Appendix).[91]

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[90] *Metaphysics* I 985 15, in Aristotle (1984).

[91] In May 1932, in his "Philosophy" course, Wittgenstein had turned toward applications of his views to the foundations of logic and mathematics, formulating the need for an explicit uniqueness rule in (quantifier-free) equational specifications of recursive definitions as a replacement for an explicit principle of mathematical induction (May 20, 1932, see Stern, Rogertsand Citron, eds. (forthcoming)). This was a genuine contribution to logic, and one that I suspect led him, in part, to teach "Philosophy for Mathematicians" in the fall (notes are in AWL and better ones in the Skinner archives). It is quite possible that Turing attended this course in either 1932 or 1932-3. Cf. Floyd (forthcoming c), and, on the recursive definition formulation, Goodstein (1945), p. 407n, von Plato (2014), Marion and Okada (unpublished).

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What Turing did in "On Computable Numbers" was to rigorize Wittgenstein's attempted philosophical generalization on the axiomatic method. For, partly indebted to Wittgenstein, he rooted his analysis of the notion of "calculation in a logic", or even of a "formal system" in a "language game", a snapshot of human calculative behavior, a person with pencil and paper living in a social world. Turing analyzed the notion of "axiomatic" or "formal" system by thinking through what we *use* such a thing *for*, by what is *done* with it. It is a massively bold, yet massively successful and concrete simplification.

While Turing was an undergraduate Wittgenstein had thematized, with "language games", the idea of a possible set of orders or commands and possible movements and reactions, possible impositions of procedures and possible steps taken with these by human beings living an active, shared life in words. The variety is most important, and when Wittgenstein discusses *Lebensform* at the beginning of the *Investigations* it is to stress just this. But this is not done with the aim of debunking the axiomatic method *per se*, much less logic itself, as if providing it with some kind of philosophical rival---for it has none.

Instead, Wittgenstein wanted to probe down into the fundamental interstices *of* logic. "To imagine a language is to imagine a form of life" (PI §19), that is, a possible structuring of life, one that shows an aspect of life, draws out procedures we have and concoct *in* life. Echoing that key remark of the *Tractatus*: form is the *possibility* of (living) structure.

Wittgenstein always wanted to show, among many other things, that logic has no basic propositions (in the sense of Euclid), requires no essential appeal to "self-evident" axioms or laws. It is not to be conceived as a science of thoughts or propositions in Frege's or Russell's senses, but rather as an activity in which we show how to construct apparent sentences and chains of reasoning that are tautologies and/or contradictions, in which we show how—as we might colorfully put it—to get to the limit of "therefore", with no premises needed at all, just a "form" of pure possibility leading to nothing but an indefinite substitutability of words, one in which the force of *particular* word choices is made to disappear, so long as one substitutes in accordance with an ordered, fixed, procedure.

Language games were conceived, beginning in the *Blue* and the *Brown Book*, anthropologically: the aura of everyday life and culture were being drawn in.[92] Here "common sense" and "ordinary phraseology" with signs and words are understood to be at work in the

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[92] Cf. Engelmann (2013) for a helpful analysis of the transitions.

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backdrop, they are part of the "reality".[93] The point was indeed to get to what Hardy and Littlewood called "gas", the use of diagrams, analogies and the ordinary to-and-fro of conversation in mathematics, and make it *part* of living mathematics itself.

But Wittgenstein also described his language-games as "experiments".[94] There are dangers in speaking only of tribes: for what was coming to matter most is that these are *possible* formations of action, not relativized meanings.[95] Wittgenstein's uses of "language games", with their stress on snipping and transporting and amalgamating and snapshotting the living variety and interlocking applications of procedures in language, imagined "forms of life" (and the "criteria" embedded in these) *replace* the notion of "common sense" with something more plastic, simpler, more evolutionary and dynamic, but also more ubiquitous, flexible and creative, more logically sophisticated.

Forms of life and imagined language games bring *life* to possibilities and necessities, characterizations of characterization itself, and they implement repeatable possible procedures or models. They model issues of difficulties that lead to genuine puzzlement: contrasts and comparisons among routines, confusion about how to shuffle parameters, shifts in aspects of uses of signs, the amalgamations of different routines, in our *perception* of possibilities and necessities. They draw out Wittgenstein's whole idea of *scaffolding*: we can put this activity up and take it down, using it to build whatever particular structure we like. It is supported by itself, and supports nothing. Its parts substitute in for one another step by step, or not (scaffolding is a kind of chaining, or binding, after all). We leave it behind when we move to forward a particular structure, a particular thing said—when the structure we seek is erected—except that its aspect is always there to *be* drawn out, unfolded, re-erected whenever we need it.

It is this agreement in measuring, weighing, gauging, voicing, and so on that is the harmony, the *Übereinstimmung*, of which Wittgenstein writes in PI §241. It is on this that not

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[93] "Common sense" and "ordinary phraseology" are not Wittgenstein's terms for this, of course: they are Turing's. Cf. Floyd (2013a, forthcoming c). However "phraseology" does occur in *The Blue Book* and in LFM, each of which Turing knew of (or likely so, in the case of *The Blue Book*). See BB, p. 69; it also appears in Wittgenstein's manuscripts and lectures around differing conceptions of numbers and mathematics, cf. e.g. (1999) MS 121, p. 76 (1939), MS 126, p 131 (1942-43); MS 127, p. 194 (1943) and (1976), pp. 18,91,98. Floyd and Mühlhölzer (unpublished) discuss this notion at length in the context of an analysis of Wittgenstein's annotations to a 1941 edition Hardy's *Course of Pure Mathematics*.

[94] BB, pp. 42, 47; This phrasing is also used in the Francis Skinner version of the *Brown Book*, now at the Wren Library, Trinity College Cambridge.

[95] Cf. Engelmann (2013) on the danger of "tribes".

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only the having of opinions, but even the very idea of "calculation in a logic", rigorously

depends.

4. *The Band of Life (Band des Lebens) (Part II, §i) and The Accepted, the Given, as*
    Aspectual (Part II, §xi)

Form is embedded, in Wittgenstein's later figure, in the stream of life: it floats there, and is

fluid. The ideal of logic as "sublime", as somehow unshakable through rigor and clarity of

compositionally fixed rules for the logical construction of propositions, ones that always operate,

that have no outside, or an outside with only "gas", is in fact suffocating: without conceiving of

an "outside" to the routine, there is no air or breath of life [*Lebensluft*, PI §103]. Without the

possibility of an "outside" embedding into everyday uses of language, any particular

compositional routine with sentences leaves us with signs that no longer live and breathe. We

deaden them with our philosophical conception. Chains are there to bind and weave with.

And yet for all of this, in Wittgenstein "logic" does comprise iterative, repeatable,

reproducible procedures, routines that begin and terminate, that are well-founded (in that they

begin *somewhere*), that are designed for a purpose, that may be broken off, amalgamated,

communicated, interwoven, and shared. There *is* simplicity. Logic is still "formal", without

being "formalist": Wittgenstein remembers Frege's animadversions against the formalists, and

takes the formal to be saturated with a world of life or sense, even if he always rejected,

following Russell, a two-dimensional distinction between *Sinn* and *Bedeutung*.

Frege was right to criticize the formalists. But he was wrong to equate the form/content

distinction with that between sign and signified.[96] Instead, he should have thought through,

Wittgenstein held, his own organic notion of an aspect, of multiple generality, something that

was crucial to his *procedures* with language in logic.[97] For the notion of *aspect*, like that of

form, is a *logical* and a *modal* notion in Wittgenstein, as I have insisted. The notion was invoked

by Frege in emphasizing the organic, multiply interlocking character of his logic's handling of

generality in comparison to e.g. Boole's. This is evinced in the multiple analyzability of,

e.g.,"Caesar crossed the Rubicon" into "x crossed the Rubicon", "x crossed y", "Θ(Caesar)", and

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[96] Cf. Frege (1981/1997), Mühlhölzer (2008).
[97] Cf. Frege's use of aspect in (1891/1997), p. 5; (1892/1997), p. 27.

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so on, determining the structure of multiple generalizations that are possible.[98] But he remained in a static world, nonetheless, when it came to the nature of the logical, so far as Wittgenstein saw. *Possibility* was banished, and with it the image of the organic idea and the gap-free chain.

The weaving idea of "criss-cross" philosophy[99] is apposite: logic rests upon no metaphysical foundations of the whole, no "glue"[100], no prior ordering or features of the world, but instead upon our activities in fashioning links in our world, one by one. We are binders, working against a universal backdrop, a whirl of life. Logic is concerned with establishing procedures and possible procedures, as well as plumbing the limits of certain kinds of procedures. Insofar it can come to be indefinitely imbricated and impressed, ubiquitously, in lives with speech and with language.

In the second, rougher, part of the *Investigations* we see that the accepted, the "given" are *Lebensformen* (PPF xi, s. 345). Here Wittgenstein is pointing toward issues thematized in *On Certainty*, drawing out the notion of "certainty" in mathematics. Forms of life are also said to be "the accepted", "the given" (PPF xi 345). They are just wherever we do begin, but it is always contestable to begin here rather than there, to break off one routine and turn to another, to take a part of a routine out and put it in elsewhere, to change a routine, alter one's sense of a face, and so on. Forms of life evince aspects that are there to be seen, altered, compared, worked with, and so on.

As part of logic in a primordial sense, this idea of the "givens" is very ancient. Euclid calls them "data".[101] As we know, this just means something given (say, a particular construction) with a request for us to show that something else *can* be done given it. This sort of "given" is not, as it became in Russell later on, an empiricist's object of immediate and infallible "acquaintance": that image breaks the chaining idea, pinning it down, limiting it. Rather a "given" is gadgetary starting point in life, something cobbled together in the "stream" of life.

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[98] Cf., again, Frege (1891/1997), p. 5.

[99] Z §447, discussed also by Diamond (2004), though not with the emphasis on *logic* that I am giving it. Contrary to what she argues, it seems to me that "What is the nature of the logical?" *is* a "Big Question" that Wittgenstein does not even if its resolution makes us see that we cannot, in another sense, answer it at one blow, but only piecemeal. So I am differing with her particular account of Wittgenstein "giving up on" "Big Questions" in the later philosophy, though not her "retail" approach to the outcome. For the origin of "crosswise" proof, which I think is the origin of this later remark, see BT §130, p. 456:
The inductive proof puts the equation together crosswise, as it were, instead of lengthwise.

[100] As Wittgenstein put it in his letter to C.K. Ogden about *Tractatus* 2.03 (1973, p. 23); quoted in Appendix below.

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In his discussion of aspects Wittgenstein draws his "Master Simile" between logical features and facial features, aspects of a human face. We become *acquainted* with these, but only by looking, responding, querying, comparing, and so on. Life and reality are to be appreciated in linkings of aspects and in reshuffling of simples. In this sense the problem of other minds is far deeper than that of the external world, as Cavell said. But, in the end, perhaps the one is reduced to the other, analogically speaking, as both are reduced to the problem of ordering, chaining, as such (binding ourselves to ourselves and to one another, in thought, deed and word). When Wittgenstein wrote that all knowledge in the end bases itself on (*gründet sich*) *acknowledgment* (*Anerkennung*), he was still, after all, echoing Frege.[102]

Forms of life, as "the given" are not given through self-evident propositions, such as those put forward as such by Moore. Nor are they proposed as actual courses of action or descriptions of what is really going on when we speak—though as Wittgenstein repeats again in this context (just as he had in PI §240), mathematicians in fact do not come to blows over what is to count as an acceptable step (PPF xi). When once they did---intuitionists to classical logicians---as a matter of fact they found a way around it, in procedures of formalization.[103] Forms (possibilities of structure) are whatever we make to *be* "given" in the course of considering and exploring, not only proposed objects of comparison, but proposed starting and juncture and stopping points, simples, accepteds (as we might call them). It does not matter, with Turing's model, *where* we begin. What matters is *that* <u>we</u> begin.

With Wittgenstein's philosophical determination to push it through rigorously, his "language game" conception provided the wherewithal for a massively fruitful and fateful generalization with respect even to everyday life. We live now, in everyday life, in a world increasingly rapidly shaped by the fact of the stored program computer. That kind of device is the realization of a particular and utterly robust conception of logic. We live in Turing's world, and bind our activities and habits more and more with their computational routines. We live in a

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[101] And so, appropriately, Kenny (2006, p.13) calls "forms of life" "the ultimate data" in W., a replacement for the simple "atoms" of the TLP.

[102] OC §378.

[103] As Gödel did, showing their connections. As a matter of fact, Turing's (1936/7) diagonal argument does *not* use negation, as the Halting argument does. Many believe the latter is what he actually gave, but it is not so. His argument is *positive*, not *negative*---as Wittgenstein clearly saw (RPP I §1096, originally penned 30 July 1947 MS 135 pp. 118ff, cf. TS 229 §1764; these are interpreted in Floyd (2012c, forthcoming c). The reason, noted by Bernays right away and spoken to in Turing's correction (1937), is that even an *intuitionist* can accept Turing's analysis of formal system and "calculation in a logic". The generality of logic itself is assured by not relying on negation: another Wittgensteinian theme.

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world of *total* mobilization and documentation: the social is part of the mapping itself, so that the device on the wall (or in one's hand, or ear, or heart) is very different from a television.[104] We live in a world where we can even see the prospect of synthetic life: artificial bindings of genes to create humans. We live in an ungrounded but massively proceduralized world. The old metaphysical picture of Aristotle cannot hold for *logic* itself, but *logic itself* shows why, and not some "miracle".

We still require an as yet unwritten philosophy for all this. Wittgenstein cannot provide us with that. But, because he was a philosopher of logic, we may do well to try to enter his ways of thinking and adapt them to our lives.

5.   *Finale: "Culture" and "Value"*

My emphasis on form-as-logical has usually not been argued for in connection with Wittgenstein's uses of "*Lebensform*". Most readers have either focussed *narrowly*—either on piecemeal "language-games" or the programming of our active behavior with words in terms of training in rules—or *widely*, thinking of *Lebensformen* as ways of life in the large, cultures, or organic internalizations of potential ways of acting in individuals.[105] The former fits the stress on variety of differing snapshots or models of language use invoked in §§19 and 23; the latter seems invited by Wittgenstein's exploration of the concepts of *hope* and *grief* in PPF i §1.

In either sort of reading the association of *Lebensform* with scaffolding and logic, with binding and weaving, with handwriting charactery, and with certainties in our proceedings that occur in PI §§240-242 and PPF xi §§345-362 is unfortunately lost. Nevertheless, we need to unbury it. For, by a remarkable philosophical transposition, Augustine's *Confessions* association between, on the one hand, the teaching and carrying on of a tradition of reasoning, or grammar, or logic (with its need for merciless drill in calculation) and, on the other hand, grief (the unending increase of the weary ways we are compelled to pass by the multiplication of suffering for the sins of the sons of Adam), has not been lost in the *Investigations*. It is as if Wittgenstein came back around to a better way to place it, analyzed it. The echo---unconscious or not---is of course to his 1930 proposed epigraph for his book *Philosophical Remarks*, published

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[104] Ferraris (2014); cf. Floyd and Katz, eds. (2016).

[105] Hunter (1968) forms a good starting point here, in distinguishing the narrower, wider, and "organic" picture of *Lebensformen*. Winch (1958/2008) is the *locus classicus* of the culture-wide idea, on which cf. Stern (2004).

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posthumously with his 1930 Foreword, one which *does* comment explicitly on culture, characterizing it in general terms. Only in Part II of the *Investigations*, by application after relentless application of the regressive method of analysis, has Wittgenstein's made Augustine's association quite rigorous, prescinding from any specific pronouncements about our "culture". Such downward looking hierarchical pronouncements by philosophy was the stuff of Russell and Schlick, a world that had already been destroyed. It was not for Wittgenstein, and it is not for us.

Wittgenstein's use of *Lebensform* is intended to *replace* an undifferentiated notion of culture, so far as I can see. The notion of *culture* does not appear in *Philosophical Investigations* at all—it was expurgated already from the *Big Typescript* and the remarks on Frazer—surely something intentional.

The uses of *Kultur* that appear fairly late, in *The Brown Book* and in RFM, are near subsequent remarks he makes that use the notion *Lebensform* to clarify. Each treats broad features of imagined or longstanding *historical* culture ("natural history"): differing uses of color terms than ours; calculation with figures in the sand, as drawn from the ancient paradigms of procedure in Babylonia.

No particular culture figures in the *Investigations* except its own, as Cavell has said. In "the darkness of this time", it's not clear there is such. The "culture of voice" in the *Investigations* explores the possibilities, not a world: it is schematic, not *bourgeois*. It lays the foundation for any *possible* state.[106]

Though Wittgenstein writes about *Kultur* in letters to acquaintances, and the notion was very important to his self-conception (as in the drafted Foreword printed with PR), his philosophy is pursued without reliance on, or presuppositions about, this. His unwillingness to so rely was made quite explicit in his initial lecture at Cambridge in Michaelmas 1930. Stating that there has been a "kink" in the development of human thought, he held that the "style of thinking" has changed, the "nimbus" of greatness has been lost to philosophy since the time of Galileo, for a "method" has been found (he must mean here science or, more generally, analysis). Now "for the first time" there can be "skillful" philosophers, but the price is a "general tendency" of the

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[106] Cf. Wittgenstein's remark about Ramsey being a "bourgeois" thinker concerned to shore up the present state (e.g., the laws of classical logic), rather than, like Wittgenstein, attempting to secure the foundations of any possible one (MS 112 70v, 1 November 1931). This remark occurs in the midst of very interesting discussions of the boundaries of mathematics (using an Euler diagram) and a quote from Lichtenberg: „ Unsere ganze Philosophie ist Berichtigung des Sprachgebrauchs, also, die Berichtigung einer Philosophie, und zwar der allgemeinsten."

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age to "take away possibilities of expression: which is characteristic of an age without a culture" (WCL 5:2, p. 107-8).

It is interesting that even here he associates the presence of a "culture" with "*possibilities of expression*": not everything may be said or expressed or meant at any arbitrary time, but what would be central to the work of logic (of philosophy) are the modalities, not the facts *per se*. In particular, he went on, the "enormous difference between philosophy and [the] sciences makes [a] difficulty" for us in our time. This can only have been horrifically reinforced in 1945, with the opening of the camps and the dropping of atomic bombs on Japanese civilians.

However: just as to declare that one is apolitical is to *be* political, so to eliminate the concept of *Kultur* as a working term is to pursue *Kultur*. One might add that to declare oneself to be a-rational in discussion of logic might be the most rational approach of all. My point here is that culture cannot be used without loss to gloss Wittgenstein's idea of *Lebensform*, it is very much the other way around. In fact, if I am anywhere near right in my account, it was his student Turing who opened Wittgenstein up to newly endless possibilities of self-expression, and---perhaps---our culture too.

As to "value": *Wert* does not appear in the *Investigations* either, and "normative" only once, in the reminiscence about Wittgenstein's conversation with Ramsey about the idea of logic as a "normative science" (PI §81). This expresses a debt to Ramsey's pragmatic, Peircean philosophy of logic, not a criticism.[107] The contrast here is that Ramsey wanted to investigate the *actual* state of things, beliefs, and persons (he was a *bourgeois* thinker), whereas Wittgenstein wanted to try to schematize and carry through an investigation of all of their *possible* orderings. It is quite important to stress that *Lebensformen* do *not* have to do with "value" as such. As I have put it elsewhere, aspects (visible and communicable, reproducible features (of form), in living arrangements (configurations) we have put together) are *orthogonal* to value. They may intersect with it, be bound to or interlocked or interwoven with it, and surely aspects, once fashioned and seen, may make claims on us. But *aspects* as such have a moment of schematicity, just as logic does.[108]

In Cavell's *The Claim of Reason* (1999) progress was made on the notion of *Lebensform*: in his hands it is neither treated narrowly as a "language-game" nor widely as a whole human

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[107] Here I agree with Missak (2016).

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backdrop of agreement holding anyway among those who speak, nor as a culture. Instead, Cavell leans on and develops the idea of "forms of life" with more sophistication, subtlety, and depth. He associates the notion with dailiness and criteria: shared and communicable aspects of our ordinary lives and typical procedures with words (criteria), and, most of all, with the "ordinary" understood as a field of philosophical departure and return, a twin of skepticism. Cavell thereby brings the broad notion of "logic" back to the *Investigations*. The "ordinary", the *simple*, as I have called it here, is colorful and dynamic, contested and accepted, an open and ubiquitous field for life, for philosophy. Cavell also stresses the need for an "outside" as a place of *Luftleben*, life's breathing air, its environment in utterance as well as procedure: what is jettisoned is a certain *conception* of "gas", an impossible view, *Luftgebäude*. His later idea of a culture of "voicing" inscribed into the *Investigations* (and, as I have said, further articulated by Sandra Laugier (2015)) gives us the right orientation for addressing culture such as it is or might be from the perspective of *Philosophical Investigations*.

As I see it, we may say that Wittgenstein's "forms of life" as voices, orchestrations of possible forms of life, or journeyings of voicing with aspects, constitute the necessary outcome of thinking through the idea of what, in general, calculation-in-a-logic *is*, a proper and inevitable foundational setting, a logical gloss on the essence of essence, the nature of the logical, *as* such.

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[108] Compare WVC 116 on "value".

Appendix

The Regressive Method, Hobbes and Euclid (Stillwell (2010), 2.1, p. 18):

He was 40 years old before he looked on Geometry; which happened accidentally. Being in a Gentleman's Library, Euclid's Elements lay open, and 'twas the 47 El. libri I. He read the Proposition. By G——sayd he (he would now and then sweare an emphaticall Oath by way of emphasis) this is impossible! So he reads the Demonstration of it, which referred him back to such a Proposition; which proposition he read. That referred him back to another, which he also read . . . that at last he was demonstratively convinced of that trueth. This made him in love with Geometry.

Euclid: Elements I.5 and Aspect-Shifting

Dodgson, *Euclid and his Modern Rivals*, p.47:

To make proof complete, it is necessary to point out that, owing to the original coincidence of the triangles, this same `left-hand angle of the second' is also equal to the 'left-hand angle of the first' and then, and not till then, we may conclude that the base angles of the first triangle angle are equal. This is the full argument, strictly drawn out. The Modern books on Geometry often attain their much-vaunted brevity by the dangerous process of omitting links in the chain...
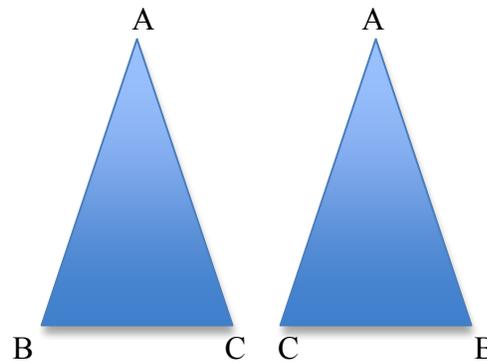
Proposition 5

For isosceles triangles, the angles at the base are equal to one another, and if the equal sides are produced then the angles under the base will be equal to one another.
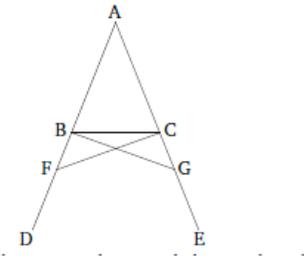
Tractatus 2.03:

> In a state of affairs objects fit into one another like the links of a **chain**.

Wittgenstein, Letters to CK Ogden p. 23:

> Here instead of "hang one on another" it should be "hang one in another" as the links of a chain do! The meaning is that there isn't anything third that connects the links but that the links themselves make connexion with one another. So if "in" in this place is English please put it there. If one would hang on the other they might also be glued together.

Tractatus 4.2ff:

> 4.2 The sense of a proposition is its agreement and disagreement with the possibilities of the existence and non-existence of the atomic facts.
> 4.21 The simplest proposition, the elementary proposition, asserts the existence of an atomic fact.
> 4.211 It is a sign of an elementary proposition, that no elementary proposition can contradict it.
> 4.22 The elementary proposition consists of names. It is a connexion, a concatenation [chaining, ***Verkettung***], of names.
> 4.221 It is obvious that in the analysis of propositions we must come to elementary propositions, which consist of names in immediate combination.
> > The question arises here, how the propositional connexion comes to be.
> 4.2211 Even if the world is infinitely complex, so that every fact consists of an infinite number of atomic facts and every atomic fact is composed of an infinite number of objects, even then there must be objects and atomic facts.

BT §131 The Recursive Proof Doesn't Reduce the Number of Fundamental Laws

> So here we don't have a case where a group of fundamental laws is proved by a group with fewer terms while everything in the proofs remains the same. (Just as in a system of fundamental concepts, nothing is altered in the later development by using definitions to reduce the number of fundamental concepts.)
> (And by the way, what a fishy analogy that, between "fundamental laws" and "fundamental concepts"!)….
> ⋯Let's imagine a **chain**: it consists of links, and it's possible to replace each such link by two smaller ones. Then the connection that the chain makes can be made entirely by the small links instead of by the large ones. But we could also imagine every link in the chain consisting of, say, two parts, each shaped like half a ring, which together formed the link, but could not be used individually as links.
> Now it would mean completely different things to say, on the one hand: The connection made by the large links can be made entirely by small links – and on the other: This connection can be made entirely by large half-links. What's the difference?
> The one proof replaces a chain with large links by a chain with small links, the other shows how one can fabricate the (old) large links from more than one part.
> The similarity, as well as the difference, between the two cases is clear to see.
> The comparison between the proof and the chain is, of course, a *logical* comparison, and therefore a completely exact expression of what it illustrates.

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Abbreviations of Wittgenstein's Works:

AWL   Wittgenstein (1979b)
BB   Wittgenstein (1965)
BT   Wittgenstein (2005)
EPB   Wittgenstein (1969)
LFM   Wittgenstein (1976/1989)
NB   Wittgenstein (1979a)
PR   Wittgenstein (1980a)
OC   Wittgenstein (1974)
PR   Wittgenstein (1980a)
PI   Wittgenstein (2009)
RFM   Wittgenstein (1978)
RPP I Wittgenstein (1980b)
RPP II Wittgenstein (1980c)
WA   Wittgenstein (1993-1997)
WCL   Stern, Rogers and Citron eds. (forthcoming)
WVC   Wittgenstein and Waismann (1979)

Z   Wittgenstein (1981)

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